Fractions

I. Fractions

a. <u>Definition</u>: A fraction (also called a rational number) is a number that represents the quotient (or division) of two integers.

b. $\dfrac{a}{b} = \dfrac{\text{Numerator}}{\text{Denominator}}$

c. The *denominator* tells how many equal parts there are.

d. The *numerator* tells how many of these parts are taken or used.

e. <u>Simplest form</u>: A fraction is said to be written in *simplest form* when there are no common factors in the numerator and denominator. The fraction has been reduced completely.

II. Zero

a. $\dfrac{0}{c} = 0$ where $c \neq 0$.   In other words, zero in the numerator of a fraction yields zero for an answer as long as the denominator does not equal zero.

b. $\dfrac{c}{0} =$ undefined.   In other words, division by zero is impossible!

$\dfrac{0}{0} =$ undefined.

III. Mixed Numbers & Improper Fractions

a. You can change a mixed number to an improper fraction:

b. $2\dfrac{3}{8} = 2\dfrac{3}{8} = \dfrac{(8 \times 2) + 3}{8} = \dfrac{19}{8}$

<u>Steps:</u>
1. Multiply the denominator by the whole number.
2. Add to the numerator.
3. The product becomes the new numerator.
4. Denominator remains the same.

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IV. Multiplication and Division of Fractions

a.   1. $\dfrac{2}{3} \times \dfrac{7}{5} = \dfrac{14}{15}$ Multiply across the numerator and across the denominator.

   2. $\dfrac{2}{\cancel{3}_{1}} \times \dfrac{\cancel{6}^{2}}{5} = \dfrac{4}{5}$ You may cancel before multiplying.

b.   1. $\dfrac{2}{3} \div \dfrac{7}{5} = \dfrac{2}{3} \times \dfrac{5}{7} = \dfrac{10}{21}$ To divide, multiply by the <u>reciprocal</u>. **DO NOT** cancel before inverting!!!

   2. $\dfrac{2}{3} \div \dfrac{19}{6} = \dfrac{2}{\cancel{3}_{1}} \times \dfrac{\cancel{6}^{2}}{19} = \dfrac{4}{19}$

V. Equivalent Fractions

a. You may multiply of divide both numerator and denominator of a fraction by the same non-zero number without changing the fraction.

   1. $\dfrac{2}{3} \times \dfrac{7}{7} = \dfrac{14}{21}$ and $\dfrac{14}{21} = \dfrac{2}{3}$ These are <u>Equivalent Fractions</u>.

   2. $\dfrac{2}{3} \div \dfrac{5}{5} = \dfrac{2}{3} \times \dfrac{5}{5} = \dfrac{10}{15}$ and $\dfrac{10}{15} = \dfrac{2}{3}$

b. Your answer to a problem may be $\dfrac{2}{5}$, by the selection of answers may look like:

   A. $\dfrac{5}{30}$ B. $\dfrac{13}{30}$ C. $\dfrac{10}{30}$ D. $\dfrac{12}{30}$ E. $\dfrac{18}{30}$

   The correct answer is D because $\dfrac{12}{30} = \dfrac{2}{5}$.

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VI. **<u>Addition and Subtraction of Fractions</u>**
   a. You may add or subtract fractions with the <u>same denominator</u> by adding (or subtracting) the numerators and putting this answer over the common denominator.

   Ex) $\dfrac{2}{5} + \dfrac{1}{5} = \dfrac{3}{5}$ 　　　　　　　 Ex) $\dfrac{7}{10} - \dfrac{1}{10} = \dfrac{6}{10} = \dfrac{3}{5}$

   b. To add (or subtract) fractions with different denominators you must first find the <u>Least Common Denominator</u> (LCD) and convert each fraction to an equivalent fraction whose denominator is the LCD.
      1. <u>Definition</u>: The LCD is the smallest number that each denominator will divide into evenly. (The LCD is the smallest multiple that the given denominators have in common.)

<u>Example – Finding Common Denominators</u>:
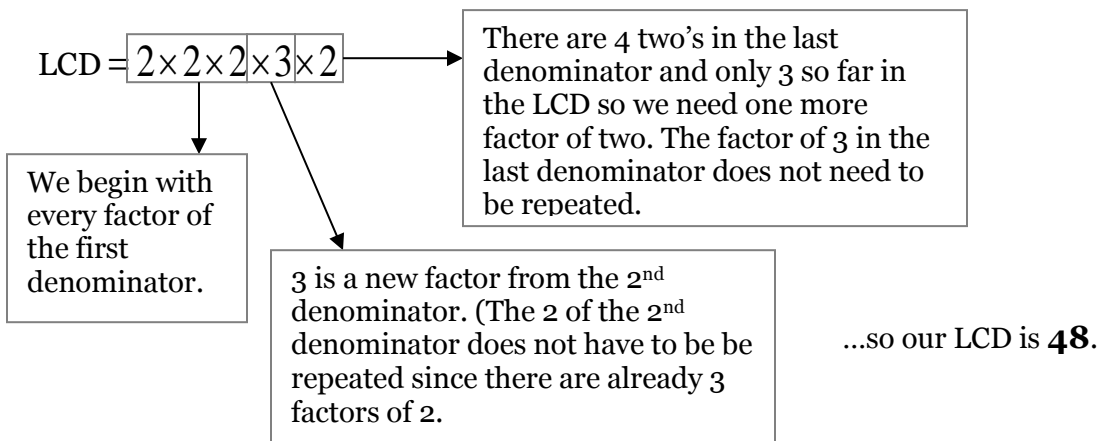
$$\frac{1}{8} + \frac{5}{6} - \frac{7}{48}$$

Examine each of the given denominators and break them down into their prime factors:

$8 = 2 \times 2 \times 2$

$6 = 2 \times 3$

$48 = 2 \times 2 \times 2 \times 2 \times 3$

The LCD is equal to <u>every</u> factor of the first denominator multiplied by any new factor that appears in a subsequent denominator.

$\text{LCD} = \boxed{2 \times 2 \times 2 \times 3 \times 2}$

There are 4 two's in the last denominator and only 3 so far in the LCD so we need one more factor of two. The factor of 3 in the last denominator does not need to be repeated.

We begin with every factor of the first denominator.

3 is a new factor from the 2nd denominator. (The 2 of the 2nd denominator does not have to be be repeated since there are already 3 factors of 2.

…so our LCD is **48**.

Now that we have our LCD, let's finish the problem:

Change each of the given fractions to an equivalent fraction whose denominator is the LCD.

$$\frac{1}{8} + \frac{5}{6} - \frac{7}{48} = \frac{6}{48} + \frac{40}{48} - \frac{7}{48}$$

Add the equivalent fractions and reduce the answer.

$$\frac{6}{48} + \frac{40}{48} - \frac{7}{48} = \frac{39}{48} = \frac{13}{16}$$

Practice Problems:

1. $\dfrac{3}{4}(20) =$

2. $\dfrac{5}{8} + \dfrac{1}{8} =$

3. $\dfrac{5}{8} \div \dfrac{1}{8} =$

4. $\dfrac{1}{2} + \dfrac{1}{3} =$

5. $\dfrac{2}{5} + 2\dfrac{5}{6} =$

6. $\dfrac{5}{8} + \dfrac{3}{4} =$

7. $\dfrac{3}{4} - \dfrac{1}{2} =$

8. $4\dfrac{5}{6} - 2\dfrac{1}{3} =$

9. $3\dfrac{15}{16} - \dfrac{7}{18} =$

10. $\dfrac{9}{7} - \dfrac{3}{35} =$

11. $\dfrac{2}{3} \times \dfrac{3}{8} =$

12. $\dfrac{3}{5} \times \dfrac{1}{3} \times \dfrac{5}{8} =$

13. $3\dfrac{1}{4} \times 8 =$

14. $\dfrac{2}{3} \div \dfrac{4}{9} =$

15. $\dfrac{2}{3} \div 4 =$

16. $\dfrac{15}{16} \div \dfrac{3}{4} =$

17. $\dfrac{11}{12} \div \dfrac{21}{4} =$

18. $7 \div \dfrac{1}{2} =$

19. $\dfrac{14}{0} =$

20. $\dfrac{0}{14} =$

21. $\dfrac{1}{13} \div 0 =$

22. $0 - \dfrac{1}{13} =$

23. $\dfrac{4}{3} + \dfrac{3}{10} - \dfrac{5}{6} =$

Answers to Fractions:

1. 15

2. $\dfrac{3}{4}$

3. 5

4. $\dfrac{5}{6}$

5. $3\dfrac{7}{30}$

6. $\dfrac{11}{8}$ or $1\dfrac{3}{8}$

7. $\dfrac{1}{4}$

8. $2\dfrac{1}{2}$

9. $3\dfrac{79}{144}$

10. $\dfrac{6}{5}$ or $1\dfrac{1}{5}$

11. $\dfrac{1}{4}$

12. $\dfrac{1}{8}$

13. 26

14. $1\dfrac{1}{2}$

15. $\dfrac{1}{6}$

16. $\dfrac{5}{4}$ or $1\dfrac{1}{4}$

17. $\dfrac{11}{63}$

18. 14

19. Undefined.

20. 0

21. Undefined.

22. $-\dfrac{1}{13}$

23. $\dfrac{24}{30} = \dfrac{4}{5}$